

I. Review Techniques of Integration:

Integration by Parts, Partial Fractions etc.

II Improper Integrals

Types of Improper Integrals

Interval of integration includes positive and/or negative infinity

Function becomes infinite somewhere on the interval of integration

Convergence

Direct Comparison Test

Limit Comparison Test

III. Sequences and Subsequences

Bounded Nondecreasing Sequence Thm

Sandwich Thm

Continuous Function Thm

Use of L'Hopital's Rule

Common Limits from Table 8.1 (pg. 625)

What is a subsequence?

$\{s\}$ a convergent sequence with limit $L \Rightarrow$ every subsequence $\{q\}$ of $\{s\}$ converges to L

$\{q\}$ a divergent subsequence of sequence $\{s\} \Rightarrow \{s\}$ diverges

$\{q\}$ and $\{p\}$ convergent subsequences of sequence $\{s\}$ that converge to *different* limits $\Rightarrow \{s\}$ diverges

$\{p\}$ a convergent subsequence of $\{s\} \Rightarrow \{s\}$ is convergent (read this, does **not** imply)

IV. Infinite Series

General Categories

Series of Numbers

Important special case: geometric series and its limit

Series of functions of a variable

Power series

Taylor series

Special case: Maclaurin Series (expanded about zero)

Characteristics of Series Elements for series of numbers,

or series of functions of a variable when the variable is taken to assume a particular value or set of values

Nonnegative series

Telescoping series

Alternating series

Tests for Convergence/Divergence

Nth term test for *Divergence*: General Series

Integral Test: Nonnegative Series

Direct Comparison Test: Nonnegative Series

Limit Comparison Test: Nonnegative Series
Ratio Test: Nonnegative Series
Nth Root Test: Nonnegative Series
Leibnitz Test for Conditional Convergence: Alternating Series
Use tests for nonnegative series on the absolute value of terms
of series to test for absolute convergence: Alternating Series
Absolute Conv. \Rightarrow Conditional Conv: Alternating Series
Conditional Conv. \nRightarrow Absolute Conv: Alternating Series
Absolute Conv \Rightarrow limit of series does not vary when
terms are rearranged
Convergence Thm for Power Series
How to Test for Convergence of Power Series
Radius and Interval of Convergence: Power Series

Error Estimation

Alternating Series Estimation Thm
Taylor Remainder Formula: for Taylor/Maclaurin Series
Taylor Remainder Estimation Thm: for Taylor/Maclaurin Series
Important special case: finding bound for error of linearization

Applications of Power Series

Term by Term Intgegration and Differentiation
Obtaining new Taylor Series from old, ie. $T(xe^x) = xT(e^x)$
Solving Differential Equations and Initial Value Problems
Evaluating Nonelementary Integrals and Indeterminate Forms

V. **Polar Coordinates**

Going back and forth between polar and cartesian coordinates
Multiple equivalent sets of polar coordinates for a given point
Graphing and finding intersections in polar coordinates

VI. **Vector Geometry**

Direction and Magnitude components of vectors
Dot Products, relation to orthogonality and angle between vectors
Vector Projection, scalar component
Cross Products, magnitude relation to area of spanned parallelogram
Cross Products, direction relation to normal direction of spanned plane
Equation for a plane
Applications of Dot and Cross Products
Distance from Point to Line
Distance from Point to Plane
Intersection of a Line and a Plane
Angles between planes, using normal directions
Line parallel or perpendicular to plane through point(s)
Line through Point in a given Direction

Line through two Points

Volume of Paralleliped

VII. **Vector Valued Functions and Space Curves**

Position Vector & Space Curves

Continuity, Differentiability,

Velocity Vector

magnitude of as "speed"

integral of magnitude as arclength

Acceleration Vector

Integrating Vector-valued Functions of a Single Variable

Arclength, see Velocity Vector

VIII. **Multivariable Functions and Partial Derivatives**

Domains, Ranges

Open, Closed, Bounded, Unbounded Subsets of the Plane

Limits & Continuity

Two-Path Test for Nonexistence of a Limit

Level Curves and Level Surfaces

Partial Derivatives, definition

Critical Points

Classification Using Second Partial

Maxima and Minima on Closed, Bounded Regions

Linearization of Functions of Two Variables

Bounding the Error of the Linearization

(a generalization of Taylor Remainder Thm)

Predicting change with differentials

The Chain Rule

Implicit Differentiation

VIII. **Gradients, Directional Derivatives and Tangent Planes**

Gradient as normal to level curve or level surface of function

Gradient as direction of maximum increase of function

magnitude as rate of maximal increase

negative gradient as direction of maximal decrease of function

negative magnitude as rate of maximal decrease

Directional Derivative

computing, interpreting

magnitude always less than or equal to magnitude of gradient

Obtaining Tangent Lines and Tangent Planes from Gradient

IX. **Double Integrals**

Region of Integration

Fubini's Thm

Double Integrals as Volumes

Double Integrals as Areas
Averages, Centroids
Polar Form of Double Integrals
 Transforming Intervals of Integration
 from Cartesian to Polar Coordinates
 Remembering that $dydx \rightarrow r dr d\theta$

X. Triple Integrals

Finding Limits of Integration
Fubini's Thm
Triple Integrals as Volumes
Averages using Triple Integrals
Triple Integrals in Cylindrical Coordinates
 Going back and forth between Cylindrical and Rectangular Coords
 Going back and forth between Cylindrical and Spherical Coords
Triple Integrals in Spherical Coordinates
 Going back and forth between Spherical and Rectangular Coords
 Going back and forth between Spherical and Cylindrical Coords

General Info to Bring With You

Taylor expansions for common series: $\cos(x)$, $\sin(x)$, $\ln(x)$, $\exp(x)$ etc.
Standard Trig Identities