# Partial Solutions to Homework 7 for MATH 336

## 2.5

The codewords are all linear combinations of the rows of the generator matrix. Therefore, $C = \{00000, 10110, 01011, 11101\}$. The cosets of $C$ are:

$$\begin{aligned}
00000 + C &= \{00000, 10110, 01011, 11101\}\\
10000 + C &= \{10000, 00110, 11011, 01101\}\\
01000 + C &= \{01000, 11110, 00011, 10101\}\\
00100 + C &= \{00100, 10010, 01111, 11001\}\\
00010 + C &= \{00010, 10100, 01001, 11111\}\\
00001 + C &= \{00001, 10111, 01010, 11100\}\\
11000 + C &= \{11000, 01110, 10011, 00101\}\\
10001 + C &= \{10001, 00111, 11010, 01100\}
\end{aligned}$$

Notice that we could have instead chosen 00101 as the coset leader of the second last row and 01100 as the coset leader of the last row. The standard array is then:

| | | | |
|---|---|---|---|
| 00000 | 10110 | 01011 | 11101 |
| 10000 | 00110 | 11011 | 01101 |
| 01000 | 11110 | 00011 | 10101 |
| 00100 | 10010 | 01111 | 11001 |
| 00010 | 10100 | 01001 | 11111 |
| 00001 | 10111 | 01010 | 11100 |
| 11000 | 01110 | 10011 | 00101 |
| 10001 | 00111 | 11010 | 01100 |

11111 is in the row with coset leader 00010, so we decode it as $11111 - 00010 = 11101$. (Alternatively, 11101 is the codeword in the first row of the column containing 11111.) 01011 is a codeword, so we decode it as itself.

(a) If the error 11000 occurs in the codeword 10110 during transmission, then we will receive 01110, which will be correctly decoded as 10110. (b) If the error 10010 occurs in the codeword 10110 during transmission, then we will receive 00100, which will be incorrectly decoded as 00000.

## 2.18

We will first show that if a binary code can correct all single errors, then any parity check matrix for the code has distinct nonzero columns. Let $C$ be a binary code that can correct all single errors. By theorem 1.3.6, $d(C) \geq 3$. By theorem 2.6.1, this implies that no single column, or pair of columns in the parity check matrix are linearly dependent. In other words, there is no all zero column, nor any two identical columns in the parity check matrix. Therefore, the parity check matrix for $C$ has distinct nonzero columns.

Now suppose that the parity check matrix $H$, for some code $C$, has distinct nonzero columns. Then, by the above reasoning, the smallest set that can be linearly dependent has size 3. By theorem 2.6.1, this implies that the $d(C) \geq 3$. By theorem 1.3.6, this implies that all single errors can be corrected, as desired.