Thesis Abstract

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The first part of this dissertation involves extending properties of the Singular Value Decomposition (SVD) to higher-dimensional arrays, or tensors. In the two-dimensional case, the SVD is particularly illuminating, since it reduces a matrix to diagonal form and reveals the rank. In the higher-order case, familiar linear algebra concepts such as rank, become complicated. Determining a closed-form solution for the rank of a general tensor is still an open problem.

We provide insight to the rank problem by presenting connections between the rank of a subclass of tensors and certain eigenvalue decompositions. The result is that for tensors of size $n \times n \times 2$, the rank is $n + k$ where k is the number of generalized complex conjugate eigenvalue pairs of the matrices forming the two faces of the tensor cube. Our proof is constructive and leads to an efficient algorithm for computation.

While rank can be determined for $n \times n \times 2$ tensors, efficient rank computation for general tensors is not yet possible. Consequently, we also present an algorithm to "compress" a tensor such that the mass is concentrated in fewer entries. Our algorithm extends the Jacobi SVD algorithm for matrices. The resulting tensor decomposition reduces a tensor to a form such that the sums of squares of the diagonals of the tensor are maximized.

The second problem in this dissertation involves solving shifted linear systems of the form $(A - \lambda I)x = b$ when A is a Kronecker product of matrices. The Schur decomposition is used to reduce the shifted Kronecker product system to a Kronecker product of quasi-triangular matrices. The system is solved using a recursive block procedure which circumvents formation of the explicit product.

Solving the shifted Kronecker product system efficiently involves using the complex Schur decomposition. While using complex arithmetic on a real problem is not always necessary, we illustrate that using complex arithmetic in this situation is critical to save computation time. We present a result that shows that when a real system is subjected to complex perturbations, the real part of the solution to the perturbed system solves a nearby *real* system.