High Performance Mathematics and its Management

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Seventy-five years ago Kurt Gödel overturned the mathematical apple cart: he proved it is not entirely deductive, and he held quite different ideas about legitimate forms of mathematical reasoning:

If mathematics describes an objective world just like physics, there is no reason why inductive methods should not be applied in mathematics just the same as in physics. (Kurt Gödel, 1951)

This lecture provides an introduction to Experimental Mathematics, its theory and its practice. I will discuss the 'Experimental Methodology' that David Bailey and I—among many others—have come to practice over the past two decades, [2, 3].[1] I will focus on the differences between *Discovering Truths* and *Proving Theorems.*

We shall explore various of the computational tools available for deciding what to believe in mathematics, and—using accessible examples—illustrate the rich experimental tool-box mathematicians now have access to, [1, 4], and which prominently includes NIST's forthcoming *Digital Library of Mathematical Functions*, viewable at `http://dlmf.nist.gov` .

In an attempt to explain how mathematicians use High Performance Computing (HPC) and what they have to offer other computational scientists. I will touch upon various *Computational Mathematics Challenge Problems* including some extraordinary identities triggered by

$$\int_0^\infty \cos(2x) \prod_{n=1}^{\infty} \cos\left(\frac{x}{n}\right) dx \stackrel{?}{=} \frac{\pi}{8}.$$

This problem set was stimulated by Nick Trefethen's relatively recent numerical *SIAM 100 Digit, 100 Dollar Challenge*,[2] which I may also mention.

References

[1] D. H. Bailey and J. M. Borwein, "Experimental Mathematics: Examples, Methods and Implications," *Notices Amer. Math. Soc.*, 52 No. 5 (2005), 502–514. [CoLab Preprint 269].

[2] Jonathan M. Borwein and David H. Bailey, *Mathematics by Experiment: Plausible Reasoning in the 21st Century*, A. K. Peters, Natick, MA, 2004. Extended Second Edition, 2008. (Now available on combined CD).

[3] Jonathan M. Borwein, David H. Bailey and Roland Girgensohn, *Experimentation in Mathematics: Computational Paths to Discovery*, A. K. Peters, Natick, MA, 2004. (Now on combined CD).

[4] D. Bailey, J. Borwein, N. Calkin, R. Girgenson, R. Luke, and V. Moll, *Experimental Mathematics in Action*, A. K. Peters, 2007.

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[1] Most resources are available at `http://ddrive.cs.dal.ca/~isc/portal`.

[2] See `www.cs.dal.ca/~jborwein/digits.pdf`.